Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 4D Pure Mathematics

Summer 2024
Question Paper Log P72803A
Publications Code 9FM0_4D_2406_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

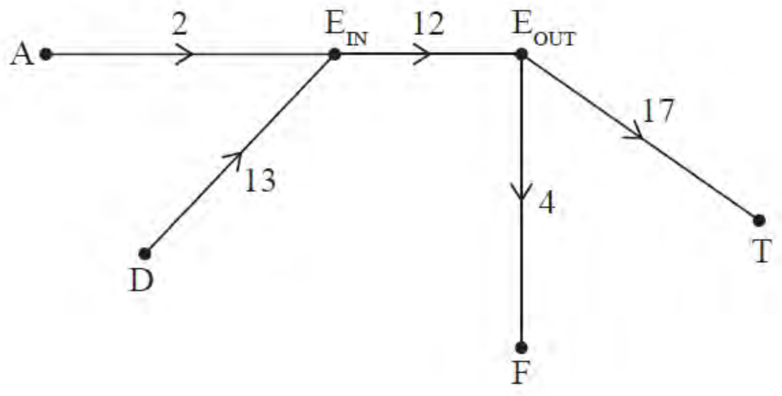
- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⊏ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	(i) $x = 10$	B1	1.1b
	(ii) $y = 7$	B1	1.1b
		(2)	
(b)	32	B1	1.1b
		(1)	
(c)	Cut $C_1 (= 13 + 12 + 12 + 11) = 48$	B1	1.1b
		(1)	
(d)	SCDFET	B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem Identification of cut through AE, DE, DT, EF and FT Value of flow = 36 It follows that flow is maximal	 M1 A1 A1	 2.1 3.1a 2.2a
		(3)	
(f)		B1	3.3
	(ii) maximum flow = 33	B1ft	2.2a
		(2)	
			(10 marks)

Notes for Question 1

(a) B1: CAO for *x*

B1: CAO for *y*

(b) B1: CAO

(c) B1: CAO

(d) B1: CAO

(e) M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs). The cut may be drawn or stated in terms of arcs but not as nodes. (Note the only saturated arc not in the cut is SB)

A1: Use appropriate process of finding a minimum cut: cut + value correct

dA1: Must have stated the value of the flow and correct deduction that the flow is maximal**.** **Must use max flow = min cut all 4 words**

dependent on previous A mark so M1 A0 A1 is not possible

(f) B1: Flows into E go to E_{IN} and flows out of E go from E_{OUT} and arc of capacity 12 from E_{IN} to E_{OUT} All arcs must have the correct arrow and capacity shown. Split node must be labelled as E_{IN} and E_{OUT} or E_1 and E_2

B1ft: Value of their maximum flow – 3

Question	Scheme	Marks	AOs
2	$u_{n+1}=A(3)^{n+1}+5(n+1)^2+1$	B1	1.1b
	$u_{n+1}=A(3)^n(3)+5(n+1)^2+1$ $\Rightarrow u_{n+1}=3(u_n-5n^2-1)+5(n+1)^2+1$	M1	1.1b
	$u_{n+1}-3u_n=-10n^2+10n+3$	A1	1.1b
		(3)	
	Alternative 1 $u_{n+1}+au_n=0$ $u_{n+1}=-au_n$ C.F. $u_n=A(-a)^n$ $-a=3 \qquad \Rightarrow \quad a=-3$	B1	
	Particular solution Try $\lambda n^2+\mu n+\nu \quad \Rightarrow \quad \lambda=5 \quad \mu=0 \quad \nu=1$ $5(n+1)^2+1-3\left(5n^2+1\right)=bn^2+cn+d$ $\Rightarrow b=-10 \quad c=10 \quad d=3$	M1 A1	
		(3)	
	Alternative 2 $u_{n+1}+au_n=3A(3)^n+5n^2+10n+6$ $+aA(3)^n+5an^2+a$	B1	
	Compares coefficients $3+a=0 \qquad \Rightarrow \quad a=-3$ $5+5a=b \qquad \Rightarrow \quad b=-10$ $c=10$ $6+a=d \qquad \Rightarrow \quad d=3$	M1 A1	
		(3)	
			(3 marks)

Notes for Question 2

B1: any correct expression for u_{n+1}

M1: eliminating A to form a first order recurrence relation containing u_{n+1} and u_n

A1: CAO for $u_{n+1} - 3u_n = -10n^2 + 10n + 3$(need not explicitly state $a = -3, b = -10, c = 10, d = 3$)

Alternative 1

B1: Considers the C.F. and deduces $a = -3$ with no other values stated

M1: Obtains P.S. and forms equation using $u_{n+1} \pm 3u_n$

A1: CAO for $u_{n+1} - 3u_n = -10n^2 + 10n + 3$(need not explicitly state $a = -3, b = -10, c = 10, d = 3$)

Alternative 2

B1: Forms the correct equation for $u_{n+1} + au_n$

M1: Attempts to compare coefficients – at least three terms seen

A1: CAO

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td>(total) demand $\neq$ (total) supply</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>Dummy</th><th>Supply</th></tr>
<tr><td>E</td><td>23</td><td>28</td><td>22</td><td>0</td><td>21</td></tr>
<tr><td>F</td><td>26</td><td>19</td><td>29</td><td>0</td><td>32</td></tr>
<tr><td>G</td><td>29</td><td>24</td><td>20</td><td>0</td><td>29</td></tr>
<tr><td>H</td><td>24</td><td>26</td><td>19</td><td>0</td><td>23</td></tr>
<tr><td>Demand</td><td>45</td><td>19</td><td>23</td><td>18</td><td></td></tr>
</table>
</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>E</td><td></td><td></td><td></td><td></td></tr>
<tr><td>F</td><td>$19-\theta$</td><td>$13+\theta$</td><td></td><td></td></tr>
<tr><td>G</td><td></td><td>$6-\theta$</td><td></td><td>θ</td></tr>
<tr><td>H</td><td>$5+\theta$</td><td></td><td></td><td>$18-\theta$</td></tr>
</table>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>E</td><td>21</td><td></td><td></td><td></td></tr>
<tr><td>F</td><td>13</td><td>19</td><td></td><td></td></tr>
<tr><td>G</td><td></td><td></td><td>23</td><td>6</td></tr>
<tr><td>H</td><td>11</td><td></td><td></td><td>12</td></tr>
</table>
Exiting cell is GB
<table>
<tr><td></td><td></td><td>23</td><td>16</td><td>19</td><td>-1</td></tr>
<tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>0</td><td>E</td><td>X</td><td>12</td><td>3</td><td>1</td></tr>
<tr><td>3</td><td>F</td><td>X</td><td>X</td><td>7</td><td>-2</td></tr>
<tr><td>1</td><td>G</td><td>5</td><td>7</td><td>X</td><td>X</td></tr>
<tr><td>1</td><td>H</td><td>X</td><td>9</td><td>-1</td><td>X</td></tr>
</table>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>E</td><td></td><td></td><td></td><td></td></tr>
<tr><td>F</td><td>$13-\theta$</td><td></td><td></td><td>θ</td></tr>
<tr><td>G</td><td></td><td></td><td></td><td></td></tr>
<tr><td>H</td><td>$11+\theta$</td><td></td><td></td><td>$12-\theta$</td></tr>
</table>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>E</td><td>21</td><td></td><td></td><td></td></tr>
<tr><td>F</td><td>1</td><td>19</td><td></td><td>12</td></tr>
<tr><td>G</td><td></td><td></td><td>23</td><td>6</td></tr>
<tr><td>H</td><td>23</td><td></td><td></td><td></td></tr>
</table>
Entering cell is FD and exiting cell is HD
</td><td>M1
A1
M1
A1
M1
A1</td><td>2.1
1.1b
1.1b
1.1b
1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(d)</td><td>(£)1882</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
</table>

<table>
<tr>
<td>**(e)**</td>
<td>
<table>
<tr><td></td><td></td><td>23</td><td>16</td><td>17</td><td>− 3</td></tr>
<tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>0</td><td>E</td><td>X</td><td>12</td><td>5</td><td>3</td></tr>
<tr><td>3</td><td>F</td><td>X</td><td>X</td><td>9</td><td>X</td></tr>
<tr><td>3</td><td>G</td><td>3</td><td>5</td><td>X</td><td>X</td></tr>
<tr><td>1</td><td>H</td><td>X</td><td>9</td><td>1</td><td>2</td></tr>
</table>
</td>
<td>M1
A1</td>
<td>2.1
1.1b</td>
</tr>
<tr>
<td></td>
<td>All II are non-negative so solution is optimal</td>
<td>A1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>**(3)**</td>
<td></td>
</tr>
<tr>
<td colspan="4">**(12 marks)**</td>
</tr>
<tr>
<td colspan="4">**Notes for Question 3**</td>
</tr>
<tr>
<td colspan="4">**(a) B1:** CAO (or to make demand = supply or because (total) supply > (total) demand (oe))
Accept e.g. A dummy demand of 18 is needed to meet supply
or there is a total of 105 supply but only 87 demand
(b) B1: CAO Check 18 in demand row
(c) M1: A valid route, only one empty square (GD) used, θs balance
A1: Correct route, up to an improved solution (seven numbers no zeros)
M1: Finding 8 shadow costs and 9 improvement indices
A1: Shadow costs and II correct (alternatives columns 0 -7 -4 -24 rows 23 26 24 24)
M1: A valid route, their most negative II chosen, only one empty square used, θs balance
A1: CSO (for part (c)) so all previous marks in this part must have been awarded – including exiting cells (GB and HD) and entering cell (FD) stated correctly (seven numbers no zeros)
(d) B1: CAO
(e) M1: Finding 8 shadow costs and all 9 improvement indices (or 8 SC and at least 1 negative II)
A1: CAO for shadow costs and the 9 improvement indices (alternatives columns 0 -7 -6 -26 rows 23 26 26 24)
A1: CSO (for part (e)) + reason + optimal) accept positive instead of non-negative</td>
</tr>
</table>

Question	Scheme	Marks	AOs
4(a)	(i) Subtracting each entry from a constant value e.g. ≥ 77 to convert from maximisation problem to minimisation	B1	3.5c
	Add a sufficiently large number (> 12) to cells BQ and DR	B1	1.1b
	(ii) e.g. $\begin{bmatrix} 12 & 5 & 8 & 2 \\ 6 & 100 & 9 & 12 \\ 7 & 8 & 4 & 0 \\ 4 & 7 & 100 & 6 \end{bmatrix}$	B1	1.1b
		(3)	
(b)	Let x_{ij} be 0 or 1 $\begin{cases} 1 & \text{if worker } (i) \text{ does task } (j) \\ 0 & \text{otherwise} \end{cases}$	B1	3.3
	where $i \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$ and $j \in \{\text{P}, \text{Q}, \text{R}, \text{S}\}$	B1	2.5
	minimise $12x_{\text{AP}} + 5x_{\text{AQ}} + 8x_{\text{AR}} + 2x_{\text{AS}} + 6x_{\text{BP}} + \text{'100'}x_{\text{BQ}} + 9x_{\text{BR}} + 12x_{\text{BS}}$ $+ 7x_{\text{CP}} + 8x_{\text{CQ}} + 4x_{\text{CR}} + 4x_{\text{DP}} + 7x_{\text{DQ}} + \text{'100'}x_{\text{DR}} + 6x_{\text{DS}}$	M1 A1	3.3 1.1b
	Subject to $\sum x_{\text{A}j} = 1, \sum x_{\text{B}j} = 1, \sum x_{\text{C}j} = 1, \sum x_{\text{D}j} = 1$ $\sum x_{i\text{P}} = 1, \sum x_{i\text{Q}} = 1, \sum x_{i\text{R}} = 1, \sum x_{i\text{S}} = 1$	M1 A1	3.3 1.1b
		(6)	
			(9 marks)

Notes for Question 4

(a) B1: correct reasoning for how to convert from maximisation to minimisation

B1: adding a large number (at least 13) to cells BQ and DR

B1: CAO

(b) B1: defining x_{ij} correctly

B1: correct definition of the values that *i* and *j* can take

M1: Attempt at a 15 (or 16) term expression in terms of *x* (or their defined variable), coefficients 'correct', 2 'large' values included, condone 2 slips. If more than 2 errors in (a) ft exactly their table for this mark only.

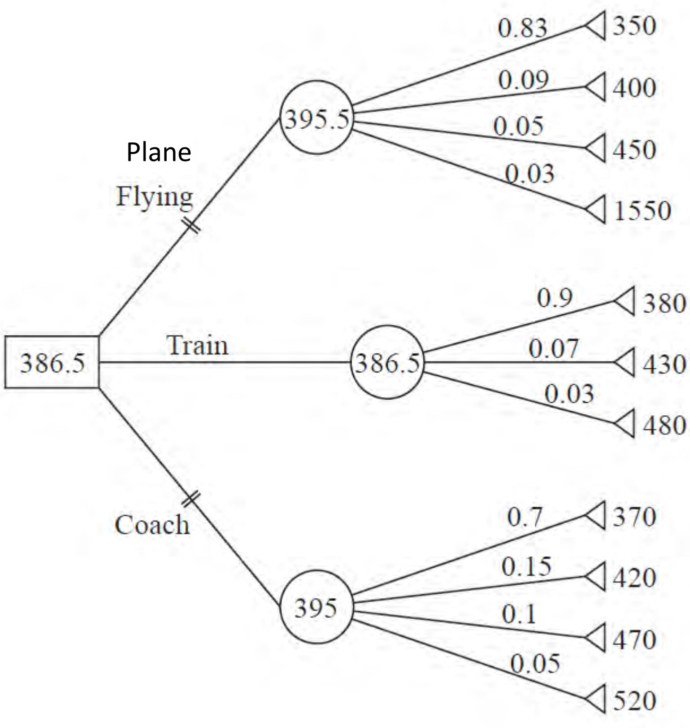
A1: CAO including **'**minimise' (or equivalent 14 term expression with 'maximise')

M1: At least four equations, each in three or four variables, unit coefficients, equal to 1 Do not accept inequalities

A1: CAO (all eight equations)

$$\text{Maximise}\quad \begin{array}{l} 65x_{AP} + 72x_{AQ} + 69x_{AR} + 75x_{AS} \\ + 71x_{BP} \qquad\qquad + 68x_{BR} + 65x_{BS} \\ + 70x_{CP} + 69x_{CQ} + 73x_{CR} + 77x_{CS} \\ + 73x_{DP} + 70x_{DQ} \qquad\qquad + 71x_{DS} \end{array}$$

Note: if maximising they must not include a large number for BQ and DR, so only 14 terms

Question	Scheme	Marks	AOs
5(a)	Note arcs must show delay time and probability  	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
		(6)	
(b)	Travel option is Train	dB1	2.2a
		(1)	
(c)	Utility values are 7.2427…, 7.2820…, 7.3282…	M1 A1	1.1b 1.1b
	Therefore, the travel option with the best expected utility is Plane	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 5
(a) Condone additional arcs seen with 0 probability (plane 3 hours, train 3 and 24 hours, coach 24 hours) **M1:** tree diagram with at least eight end pay-offs, one decision node and three chance nodes (condone missing triangles from end pay offs and incorrect shapes for decision and chance nodes) **A1:** correct structure of tree diagram with the non-zero probability 11 arcs labelled correctly (including probabilities) **M1:** at least three end-pay offs consistent with their stated probabilities (must include ticket price, cost for travel time and cost for delay – may be implied by correct values); at least eight attempted **A1:** all eleven end-pay offs correct including triangles and no incorrect extra (condone if not fully simplified) (values may be negative as costs) **M1:** all three chance nodes attempted with their probabilities **A1:** CAO for chance and decision nodes including double line through inferior options – must have the correct shapes **(b) dB1:** deduction of correct travel option (dependent on all method marks earned in (a)) **(c) M1:** At least one (of the three) utility values correct (FT their end pay offs) **A1:** At least two correct **A1:** Correct travel option (Plane) together with all three correct values (to at least 3 sf – rounded or truncated)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>Action</th><th>Dest</th><th>Value</th></tr>
<tr><td>1</td><td>H</td><td>HT</td><td>T</td><td>2*</td></tr>
<tr><td></td><td>I</td><td>IT</td><td>T</td><td>3*</td></tr>
<tr><td></td><td>J</td><td>JT</td><td>T</td><td>4*</td></tr>
<tr><td>2</td><td>F</td><td>FH</td><td>H</td><td>5 + 2 = 7*</td></tr>
<tr><td></td><td></td><td>FI</td><td>I</td><td>4 + 3 = 7*</td></tr>
<tr><td></td><td></td><td>FJ</td><td>J</td><td>6 + 4 = 10</td></tr>
<tr><td></td><td>G</td><td>GH</td><td>H</td><td>5 + 2 = 7</td></tr>
<tr><td></td><td></td><td>GI</td><td>I</td><td>3 + 3 = 6*</td></tr>
<tr><td>3</td><td>C</td><td>CF</td><td>F</td><td>3 + 7 = 10*</td></tr>
<tr><td></td><td></td><td>CG</td><td>G</td><td>4 + 6 = 10*</td></tr>
<tr><td></td><td>D</td><td>DF</td><td>F</td><td>3 + 7 = 10</td></tr>
<tr><td></td><td></td><td>DG</td><td>G</td><td>1 + 6 = 7*</td></tr>
<tr><td></td><td>E</td><td>EG</td><td>G</td><td>2 + 6 = 8*</td></tr>
<tr><td>4</td><td>A</td><td>AC</td><td>C</td><td>4 + 10 = 14</td></tr>
<tr><td></td><td></td><td>AE</td><td>E</td><td>5 + 8 = 13*</td></tr>
<tr><td></td><td>B</td><td>BC</td><td>C</td><td>4 + 10 = 14</td></tr>
<tr><td></td><td></td><td>BD</td><td>D</td><td>7 + 7 = 14</td></tr>
<tr><td></td><td></td><td>BE</td><td>E</td><td>4 + 8 = 12*</td></tr>
<tr><td>5</td><td>S</td><td>SA</td><td>A</td><td>3 + 13 = 16</td></tr>
<tr><td></td><td></td><td>SB</td><td>B</td><td>2 + 12 = 14*</td></tr>
</table>

Latest time that Elena can start her journey is 7 am</td><td>B1
M1
A1
M1
A1
M1
A1ft
A1
A1
(9)</td><td>3.1a
3.1a
1.1b
1.1b
1.1b
1.1b
1.1b
1.1b
3.2a</td></tr>
<tr><td>(b)</td><td>Route: SBEGIT</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(10 marks)</td></tr>
</table>

Notes for Question 6

All M marks – must bring earlier optimal results into calculations. Ignore extra rows. Penalise lack of * only once per question.

B1: CAO for first stage

M1: Second stage completed. At least 4 rows, something in each cell.
A1: CAO for second stage exactly 5 rows

M1: Third stage completed. At least 4 rows, something in each cell.
A1: CAO for third stage exactly 5 rows

M1: Fourth stage completed. 5 rows, something in each cell.
A1ft: correct ft their optimal values from third stage
A1: CAO for fifth stage

A1: Correct latest start time in context (e.g. 7 am, 07:00, etc.)

(b)
B1: Correct route (dependent on all previous M marks)

Special Case – Maximin or Minimax

Stage	State	Action	Dest	Value	Value
1	H	HT	T	2*	2*
	I	IT	T	3*	3*
	J	JT	T	4*	4*
2	F	FH	H	Min (5, 2) = 2	Max (5, 2) = 5
		FI	I	Min (4, 3) = 3	Max (4, 3) = 4*
		FJ	J	Min (6, 4) = 4*	Max (6, 4) = 6
	G	GH	H	Min (5, 2) = 2	Max (5, 2) = 5*
		GI	I	Min (3, 3) = 3*	Max (3, 3) = 3
3	C	CF	F	Min (3, 4) = 3*	Max (3, 4) = 4*
		CG	G	Min (4, 3) = 3*	Max (4, 5) = 5
	D	DF	F	Min (3, 4) = 3*	Max (3, 4) = 4*
		DG	G	Min (1, 3) = 1	Max (1, 5) = 5
	E	EG	G	Min (2, 3) = 2*	Max (2, 5) = 5*
4	A	AC	C	Min (4, 3) = 3*	Max (4, 4) = 4*
		AE	E	Min (5, 2) = 2	Max (5, 5) = 5
	B	BC	C	Min (4, 3) = 3*	Max (4, 4) = 4*
		BD	D	Min (7, 3) = 3*	Max (7, 4) = 7
		BE	E	Min (4, 2) = 2	Max (4, 5) = 5
5	S	SA	A	Min (3, 3) = 3*	Max (3, 4) = 4*
		SB	B	Min (2, 3) = 2	Max (2, 4) = 4*

B1 M1 A0 M1 A0 M1 A0 A0 A0 B0 - Max 4/10

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7(a)</td><td>Row minima: – 3, – 2, – 1 (max is – 1)
Column maxima: 4, 3, 6 (min is 3)</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>Row maximin (– 1) $\neq$ Column minimax (3) (so not stable)</td><td>A1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>$\begin{pmatrix} 3 & 2 & -3 \\ 4 & -2 & 1 \\ -1 & 3 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 6 & 5 & 0 \\ 7 & 1 & 4 \\ 2 & 6 & 9 \end{pmatrix}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$V - 6p_1 - 7p_2 - 2p_3 + r = 0$
$V - 5p_1 - p_2 - 6p_3 + s = 0$
$V - 4p_2 - 9p_3 + t = 0$
$p_1 + p_2 + p_3 + u = 1$
$(P - V = 0)$</td><td>M1
A1</td><td>2.1
2.5</td></tr>
<tr><td></td><td>
<table>
<tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr>
<tr><td>r</td><td>1</td><td>– 6</td><td>– 7</td><td>– 2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>1</td><td>– 5</td><td>– 1</td><td>– 6</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>t</td><td>1</td><td>0</td><td>– 4</td><td>– 9</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>– 1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>
</td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(c)</td><td>A should play R with probability $\frac{3}{8}$, option S with probability $\frac{17}{80}$ and option T with probability $\frac{33}{80}$</td><td>B1</td><td>3.2a</td></tr>
<tr><td></td><td>Value of the game to player A is $\frac{73}{16} - 3$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>So, value of the game to player B is $-\frac{25}{16}$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>$6q_1 + 5q_2 = 4.5625$
$7q_1 + q_2 + 4q_3 = 4.5625$
$2q_1 + 6q_2 + 9q_3 = 4.5625$
$q_1 + q_2 + q_3 = 1$
or
$3q_1 + 2q_2 - 3q_3 = 1.5625$
$4q_1 - 2q_2 + q_3 = 1.5625$
$-q_1 + 3q_2 + 6q_3 = 1.5625$
$q_1 + q_2 + q_3 = 1$</td><td>M1
A1</td><td>2.1
1.1b</td></tr>
<tr><td></td><td>B should play X with probability $\frac{1}{2}$, option Y with probability $\frac{5}{16}$ and option Z with probability $\frac{3}{16}$</td><td>A1</td><td>3.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
</table>

(13 marks)

Notes for Question 7

(a) M1: Attempt to calculate row minima and column maxima – condone one error (note row max are 3, 4, 6 and column min are -1, -2, -3 so we must see where values come from)

A1: Correct reasoning that the game is not stable (accept $-1 \neq 3$) – dependent on correct row maximin and column minimax

(b) Note – a fully correct tableau implies all marks in (b)

B1: Correct augmentation – possibly implied by later working in tableau

M1: At least three equations in V, p_1, p_2, p_3 and at least one dummy variable seen (must be using columns)

A1: CAO for all four equations (possibly implied by later working in tableau)

M1: Any two (numerical in nature) row correct (ignore labelling of b.v. column)

A1: CAO

(c) B1: Correct optimal strategy in context (dependent on both M marks in (b))

M1: For $\pm\left(\frac{73}{16} \pm 3\right)$

A1: CAO

(d) M1: Attempt to set up at least three equations in q_1, q_2, q_3 using the value of the game from (c)

A1: CAO (for any three of the four correct equations)

A1: CAO in context (must have at least three correct equations)

Question	Scheme	Marks	AOs
8 (a)	aux. equation is $2m^2+5m-3=0 \Rightarrow m=\ldots$	M1	2.1
	complementary function is $A(0.5)^n+B(-3)^n$	A1	1.1b
	Particular solution try $u_n=an+b$ and substitute into recurrence relation	M1	1.1b
	$2(a(n+2)+b)+5(a(n+1)+b)=3(an+b)+8n+2$ and by comparing linear and constant terms gives $4a=8$ $9a+4b=2$	dM1	1.1b
	$(u_n=)\,A(0.5)^n+B(-3)^n+2n-4$	A1ft	1.1b
		(5)	
(b)	$A+B-4=1$ $0.5A-3B-2=k$ leading to $A=\ldots$ and $B=\ldots$	ddM1	3.4
	$(u_n=)\left(\frac{34+2k}{7}\right)(0.5)^n+\left(\frac{1-2k}{7}\right)(-3)^n+2n-4$ and setting their $\frac{1-2k}{7}=0$	dddM1	3.1a
	$k=0.5$	A1	2.2a
		(3)	
			(8 marks)

Notes for Question 8

Note mark (a) and (b) together

(a) M1: correct auxiliary equation and attempt to solve (leading to two distinct values of *m*)

A1: CAO for complementary function (accept if seen in subsequent working)

M1: correct form for particular solution and substituted $n+2,\ n+1,\ n$ into recurrence relation

dM1: compares coefficients and setting up both equations in a, b(so one equation in *a* only and one equation in *a* and *b* (although *a* may have already been found from the first equation)) – dependent on previous M mark ($a = 2$ and $b = - 4$)

A1ft: correct general solution following through their complementary function (must be their C.F. + $2n – 4$) Please award for a fully correct solution seen either here or being used in (b)

(b) ddM1: use correct initial conditions correctly to form simultaneous equations in their *A*, *B* and *k* and attempt to solve for *A* and *B* – dependent on the previous two M marks. Award this mark for

Either

- eliminating either *A* **or** *B* from the correct two equations leading to one of
  $7A = 2k + 34$ **or** $7B = 1 - 2k$ (accept any equivalent form)

Or

- eliminating both *A* and *B* from their equations using the initial conditions and obtaining expressions in terms of *k*

If correct $A = \dfrac{34 + 2k}{7}, B = \dfrac{1 - 2k}{7}$

dddM1: setting coefficient (which must be a linear expression in *k*) of their $(-3)^n$ equal to zero – dependent on the previous three M marks (and at least one root of the auxiliary equation being negative)

A1: CAO (must be from correct working)

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